MATHEMATICS IN THE SOCIAL SCIENCES

Oscar Varsavsky, 20-4-67

Introduction

The subject of scientific research is often seen today as a dynamic system-physical, biological, social (by "social" I mean pertaining to any social science)-or a part of one. To describe its evolution -with its causal and random components- ordinary language has been found as inefficient as the use of handwritting for newspapers. The appropriate language is called Mathematics, and the corresponding description, a mathematical model of the system.

The use of mathematical methods in the Social Sciences is growing at a high rate, as can be seen in the general literature and in the proliferation of specialized journals and symposia.

But in the present author's opinion, this growth is not efficiently oriented. Too much talent is being wasted in low productivity fields. This article intends to open a debate on this question, with the hope of stopping the current attitude towards Mathematics: the belief that the way to theoretical advancement in the Social Sciences lies in the use of more and more sophisticated "modern" Mathematics such as the different branches of topology, special algebras or the stability theory of non-linear differential equations.

Mathematics as the Language of Physics

The impact of mathematics on the social sciences has not been spectacular, to say the least, and compares very poorly

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with what it has been for physics. This is due in part -I believe- to a lack of understanding of what modern mathematics is and what can be expected from it. And this hinders its adaptation to the specific needs of these sciences.

It is easy to mistake mathematics for what the leading mathematicians are doing. This prevents one to see the potential possibilities of mathematics, and puts the social scientist in the passive situation of having to try the instruments which the mathematicians have already developed, instead of demanding the ones he specifically requires. This is as inefficient as trying to explore the jungle using only instruments developed for seafaring.

To support this assertion, let us review very briefly the historical perspective of present day mathematics.

Modern mathematics was born to satisfy the demands of the physicists, who needed a rigorous language to express the laws and theories they were discovering. These laws had several very special characteristics:

1 - The number of variables one dealt with simultaneously was very small (half a dozen position coordinates, temperature and conductivity, an electric and a magnetic vector, a couple of wave functions).

2 - The "local" variations of these variables were of fundamental importance (temperature in very near points, changes of position in very short intervals, compared to "nearness" and "shortness" in everyday experience). To this demand, mathematics answered with the concepts of analysis: continuity, differentiability (invented, at that, essentially by the very physicists who needed them).

3 - The variables were easily quantified, measured and handled, with good reproducibility of external conditions and results. That led to precision criteria much more exacting than those of everyday life, and promoted exact prediction as the ideal scientific goal.

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4 - Linear approximations happened to be extremely good (one can write a good physics textbook without dealing seriously with non-linear phenomena). This fact gave great importance to the search for "closed" general solutions: formulae which give the unknown value as a combination of elementary functions of the data.

5 - Physical systems were almost always studied in their equilibrium state, with less interest in the "transients" that lead to it. The observers disturb the system very little and they can use all the time they want in their observations. The experimental method works just fine. The theory is predictive, not normative. Decisions and responsibility are out of context (some examples of teleology, like in the variational principles, are avoidable curies).

These characteristics of physics forced mathematics to develop around differential equations, mainly the linear kind. Calculus was the language par excellence. That is why theoretical mathematics oriented itself towards the very infinite sets, like the real numbers, and topology was invented.

Algebra did not develop as fast, until it proved useful for analysis. Mathematical language became then almost perfect, and everything was axiomatized.

In all this development, the momentum was always given by physics. Mathematicians almost never had a theory ready when it was needed. Newton, Heisenberg, Dirac and Einstein had to forge for themselves the mathematical instruments they needed. It is not fair to say, for instance, that Heisenberg simply rediscovered matrix calculus. What mathematicians knew about infinite matrices was of very little use to quantum mechanics, and even the theory (unbounded linear operators) was developed by von Neumann a posteriori. Tensor calculus and distributions theory, were also developed after the fact. Physicists were a little luckier when they needed some algebra (as in the theory of group representations).

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Very generally, we can say that mathematics has been, up to now, the language demanded by physicists, plus the continuation of very ancient investigations about the prime numbers and the algebraic equations. The theory of these subjects has developed enormously, attaining a degree of abstraction and esoterism only comparable to that of modern art 1/ .

Social Sciences and Mathematics in the past

But to repeat, this marvelous world was oriented and set on its present course by physics alone. We shall refer to it as Orthodox Mathematics.

When social phenomena were beginning to be studied scientifically, Orthodox Mathematics was well developed already, and enjoyed enormous prestige from its successes in physics, It was only natural that the first social scientists tried to employ this instrument as they found it.

1/ So far have they advanced along this road (motivated by differential equations, geometry and algebra), so far are they now from the coastline -these thousands of intelligent explorers of "Modern Mathematics"- with no other compass than their noses, that it is time to wonder whether the course they have taken is still of any interest to science, present or future. Of course anything one discovers has some kind of interest, but if all explorers had always followed the same path, we would not know half of the world. The "homo ludens" ideology that the young mathematicians apply to their activities is, to say the least, alarming.

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It cannot be said that this attempt has been very fruitful. There are, of course, many isolated instances of interesting -even surprising- applications, but there is no single social phenomenon whose theory has advanced substantially thanks to the use of Orthodox Mathematics; at least until last war.

No wonder, since social sciences have needs quite different from physics. Let us review some of their characteristics:

1 - Each single social phenomenon involves hundreds -when not thousands- of variables, all of them important a priori, and their interconnections.

2 - Continuity in time or space is not an important concept. Nobody believes that anything essential is lost if a socio-economic system is observed only at discrete intervals. Topology seems to be artificial here.

3 - The variables are quantified only with difficulty. In general one is only interested in a finite set of different values for them, and often these values do not admit of any usual algebraic structure (order, sum, product).

4 - When quantifying is possible, it is always imperfect (measurements lack precission and are not easily reproducible). Even so, linear relations with few exceptions are not acceptable -even as first approximations-.

5 - Social systems are not studied in equilibrium. In general one is not even interested in knowing whether they have or not some kind of stability, assymptotically. If they could be represented by differential equations, only the transient states would be of any use, since the future matters the less the farther it is. Predictions are mainly instruments for decisions, and there is always someone responsible for acting, whose scarcest ressource is time.

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Is it conceivable that infinitesimal calculus may play any important role under these conditions? That "goes double" for homotopy groups, the Zermelo axiom, or differential topology.

The few isolated applications of Orthodox Mathematics only confirm this assertion. Fixed point theorems have been used to prove the existence of certain economic equilibria, and even more abstract techniques to see the equivalence of several axiomatic presentations of value and preference. Differential equations have been applied to ecological problems, and some difficult points of their stability theory in the non linear case have been used in recent economic models; and let us not forget the old Volterra integral equations of the "struggle for life". But in every case the problem has been oversimplified and it is almost impossible to give it some practical use.

A genius like von Neumann produced the well known linear economic model, which cannot be used for practical planning, and a Theory of Games which did not have to be Orthodox Mathematics, but became so right away due to the scarcity of concrete applications in Social Science (in spite of many deliberate efforts to find uses for this beautiful instrument).

Remember, on the other hand, how it was possible to assign to each quantum system -by fixed rules- a linear operator in Hilbert space, whose eigenvalues reproduced with amazing precision the energy spectrum of the system. Compared to this glove-like fit between the empirical science and its mathematical language, the applications we have recalled seem weak and artifical indeed.

I do believe that von Neumann understood this, and the work of his last years (Montecarlo and Automata) was on the right path. His death may have been a more serious setback to Social Science than is usually realized.

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Up to this point I may seem to be repeating some of the arguments usually employed in the classic controversy, to prove that cultural sciences needed a different methodology than Natural Sciences, and in particular, that Cultural Sciences could not be made rigorous or mathematised. Indeed I am, only to me, those arguments do not prove the impossibility of a rigorous science of social systems, but only the inefficiency of Orthodox Mathematics as an instrument, and show the need that the best mathematical brains start paying more attention to the specific mathematical demands of these sciences. I am trying to say that the old controversy had in fact a meaning, and that thesis and antithesis could be synthesized in the need for a new mathematics: the rigorous language of social sciences.

Most promising in this view is that amorphous school called Operations Research, and it is there that the germs of a "New Math" should be looked for. Born with the practical goal of helping in decision making, it was forced, many a time, to introduce new concepts and methods (let us mention especially the chapter on simulation). But it seems to do it shamefacedly, and most of its pioneers still take every opportunity to employ the most abstract and advanced language of Orthodox Mathematics.

To date, there is no conscious, systematic effort to escape orthodoxy.

The Mathematics of Social Science

What can we say of the New Mathematics, to be used as the language of social sciences?

1 - The large number of variables and non-linearity force us to give up the search for "closed" analytic solutions, and even for assymptotic behaviour. One has to solve particular cases, numerically. That implies that New Math will be very closely related to computers. A good Computer Mathematics would be a long step forward.

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2 - All sets to be handled are finite. At most, one could be interested in a "potential" infinity, as in natural number sequence (a computer has a fixed capacity, but one can always imagine a larger computer, with a higher overflow). Irrational numbers can be ignored: pi and the square root of two are concepts uninteresting to social science. Nor is the classical theory of numbers of interest. Combinatory is probably the most useful branch of orthodox arithmetic.
An instance of a problem that should be attacked in force: to build a theory of the partitions of a finite set with a distance-like structure, introducing concepts like class homogeneity, shape, separation, etc. Something has already been done -not by mathematicians- using the entropy information standpoint.

3 - Causal laws are qualitative and "dendritic" (by analogy with the way a neuron shoots): certain simultaneous changes of state in many variables produce certain changes of state in others. It is no help to say that this effect can be represented by a general operator from a product a finite sets to another. We need a typology of such operators, based on the adequate structures of those sets. Since they are not groups with respect to any interesting operation, we are left without the linearity concept. Sometimes there is order, and we can speak about monotony, but in general we can only say that the causal operator is "general". It seems foolish to use the modern mathematical language to say so little.

4 - The time evolution of a social system is not described by differential equations, since there is always an interval below which we do not care what happens. But it would not be correct to describe this situation by saying that difference equations substitute for differential ones, because it is not the orthodox theory of difference equations that matters, either. It is not a question of solving equations.
In fact one can almost always proceed as though the equations were already solved. Models consist of sequential computations, given explicitly. <u>They are computer programs</u>, with sequential basic instructions. (That some of these instructions may be complex subroutines does not alter the fact). The ordering of those instructions is a part of the hypothesis which constitute the model.

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It would seem that in the New Math the problem of finding the inverse of an operator will be much less important than in Orthodox Math. The goal is rather to make the operator "explicit" by obtaining the values it assumes on a sample of points from its domain and passing from this to a qualitative description in terms of some typology, that is, classifying.
Here we find another instance of inadequacy: Orthodox Mathematical Logic deals almost exclusively in propositions, and very little has been said about what is of foremost interest here, namely instructions. Sequences of instructions have their particular rules and problems, which are practically unexplored country. Recursivity theory is the nearest thing to date, but it is still far from the target.

5 - Whether quantified or not, input and output of a social system model consist of an appalling amount of data. One of the main problems is how to analyze so many data in a practical way, without having to lose weeks studying each computer run. The answer seems to lie in the field of classification, which gives a qualitative summary of the large set by partitioning it.
Parameters and equations are classified according to model sensitivity (from critical to innocuous). The variables are classified in blocks of a certain degree of mutual independence. Outputs and inputs are classified in classes $O1, \ldots, On, I_1, \ldots, In$, in the endeavour to get qualitative laws that describe the model as an operator M which transforms Il into Oj.
Biologists, psychologists, engineers, sociologists, are trying to devise good methods of automatic classification. The mathematicians have not bothered yet.

6 - The qualitative laws of systems (or rather their models), are obtained by repeated experimentation. They are empirical laws, induced from the numerical experiments run in a computer. In order to be interesting they must not refer to a single model of the system under study (that is, fixed equations and parameters) but to a whole class of models that could all be reasonably considered as alternative representations of the system. In other words, it would be a miracle if independent reserchers studying the same social system came up with identical models or representations of it. The confiability of a law about the system will be high if it is true for most of its models.

Since each class of models and inputs will have many members, the experiments will be a sample from that universe, and a theory of sampling and experimental design should be developed that answers to these exceptional characteristics mainly the many non-independent factors.
The same thing can be said about sensitivity experiments to discover crucial parameters and equations. (1)

7 - The precission of physical measurements and the simplicity of its laws made it possible to obtain from its models predictions so exact that they can be counted among the most amazing human feats. It is a pity that in the social sciences, models cannot predict nearly as exactly, in any way, but that does not mean that they are not useful. Striving for precission can defeat that usefulness.

The econometric models are a typical example of this. Most of the effort is spent in obtaining good estimations of the parameters, while only linear relations are included -or other relations similarly simple-, and many obviously important variables are excluded for lack of sufficient data to estimate their coefficients.

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(1) This appeal to Statistics is made as an aside, since it is an individual science which uses mathematics as a language, but is no more a branch of the latter than physics is. It is perhaps useful to stress this, since it is the source of some confusion.
Statistics is indispensable to Science and Philosophy, but it is not a substitute for mathematics. The latter is supposed to be an efficient language to express concepts and hypothesis, and to obtain their logical conclussions: it represents ideas by rigorous models. The former is expected to tell how much faith to put on each hypothesis (whether about a causal relationship or the value of a parameter). In order to do this, statistics uses a lot of Orthodox Mathematics (but not exactly the same kind as physics), but a social scientist should not confuse the mathematics needed by statistics with his own needs. Measure theory will not help him to formulate his ideas about how certain factors influence certain behavior.
Statistics is a necessary, but certainly not a sufficient, tool for social sciences.

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This is accepted as the only "scientific" procedure by many, because physics has forced on us the standard of a quantitative agreement with reality. We are used to thinking that if it cannot be proved that a model is a precise representation of reality it is worthless. The burden of proof is on the model-maker, and when he cannot test it with predictions, he tries very hard to test it with past history.

But take for instance the use of a social system model by a person -or rather, team- responsible for decision making. This responsible agent has to act anyway, whether he has a model to guide him or not. Indeed he always has in mind some model, some representation of the situation, on which he bases his decisions. Large though the random element usually is, there are always some causal hypothesis about the effects of the instrument variables he controls. The mathematical model is then invaluable if only as the complete, clear and rigorous representation of the model in mind, whatever degree of fitness this last has with the real world. With a mathematical model of his own ideas, the responsible actor can at least discover his inconsistencies, his gaps, and he can communicate easily with his advisors to check, complete and improve different particular aspects. It is this language he needs, incomparably superior to a literary language, which is so obscure and inefficient.

Each model would be the theory of an individual or team about the social system under study. It expresses the ideas -good or bad and always hypothetical- of that team. It need not be true to reality but only to these ideas. In this way it becomes possible to check the theory with reality.

To summarize theories, in the social sciences, do not explain reality very well, yet. A mathematical model of a theory cannot improve that situation; it can only expose it clearly. No matter how "difficult" are the mathematics used, the model can

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be no better than the theory it is expressing. Between the model and reality there always has been an intermediate link: the theory of the model maker; however in physics the theories were so good and the mathematics such a natural language for them, that one never paid attention to this three link chain: reality - ob actor server - model.

This is also why a social system is not represented by one model, but by a whole family of them, since each team studying the same social system will have its own theories about it. Those theories may differ in the number or type of variables more adequate to describe the system, in the shape of the functions that represent the causal hypothesis, or just in the estimation of the parameters.

We need, then, methods to analyse not just one operator, but a universe of them, of which we have a small sample.

A large part of what we have described as "New Math" could be characterized as the theory of many variable models, with finite domains lacking the usual algebraic structures. It is not the only new mathematical field to explore, but it is wide enough to start, and is the only one I am able to suggest, having felt the need for it.

A model in social sciences resembles much the engineer's "system". There is a three-way input: initial conditions, exogenous restrictions (boundary conditions) and controls. The output is multidimensional and feeds back in a complicated way. The system is obviously not a black box, because its mathematical structure is known, but in practice it is almost so, because that structure is so complex that very little can be deduced from it analytically.

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This intermediate position between a black box and an orthodox mathematical operator is what requires new techniques of analysis. One has a lot of information about the contents of this box, but there are no standard methods to use it.

Perhaps we need a language like that in physiology, which allows us to talk about "organs" in the system, and assign to them similar functions in different models. But we even lack a good definition of "organization"!

The world of models appears to us now as the fauna of a new planet. The first stage to study it is fully empiric-intuitive: to observe how these animals behave in different situations, in order to get some feel, and distinguish the normal from the teratologic. This is done with almost random numerical experiments.

For a second, somewhat more scientific, stage, the most adequate weapon seems to be numerical taxonomy -with its natural complement, pattern recognition- since the main job is to classify: inputs, organs (blocks of variables), outputs, and with all that, the models themselves.

Meanwhile computer mathematics has to be developed, with its finite arithmetic, its instructions logic and the explicit considerations of the time-cost factor.

We need measures of complexity and degree of organization, superior to the weak concepts of entropy-information. And the statistical theory has to be developed that tells us how to measure the confiability of the laws describing model behavior.

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